Chapter 10

1

Rotation Of A Rigid Object About A Fixed Axis

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Lecture 05

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Work And Energy In Rotational Motion

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Work in Rotational Motion

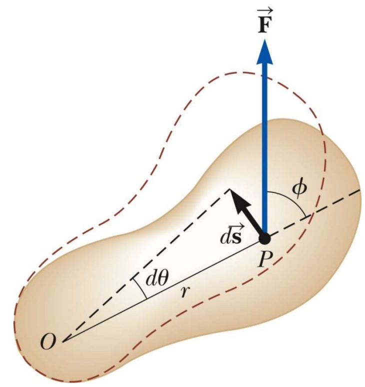
Find the work done by $\vec{F}$ on the object as it rotates through an infinitesimal distance $ds = rd\theta$.

$$dW = \vec{F} \cdot d\vec{s} = (F sin\varphi) r d\theta$$

The radial component of the force does no work because it is perpendicular to the displacement.

$$W = rF sin\varphi \int d\theta$$

$$W = \tau\theta$$



Power in Rotational Motion

The rate at which work is being done in a time interval dt is

$$P = \frac{dW}{dt} = \tau \frac{d\theta}{dt} = \tau\omega$$

- This is analogous to $P = Fv$ in a linear system.

Work-Kinetic Energy Theorem in Rotational Motion

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- The work-kinetic energy theorem for rotational motion states that:

"The net work done by external forces in rotating a symmetrical rigid object about a fixed axis equals the change in the object's rotational kinetic energy".

$$\sum W = \int_{\omega_i}^{\omega_f} I\omega \, d\omega = \frac{1}{2} I\left(\omega_f^2 - \omega_i^2\right)$$

- The rotational form can be combined with the linear form which indicates the net work done by external forces on an object is the change in its total kinetic energy, which is the sum of the translational and rotational kinetic energies.

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Rotational Motion About a Fixed Axis	**Translational Motion**
Angular speed $\omega = d\theta/dt$	Translational speed $v = dx/dt$
Angular acceleration $\alpha = d\omega/dt$	Translational acceleration $a = dv/dt$
Net torque $\Sigma\tau_{\text{ext}} = I\alpha$	Net force $\Sigma F = ma$
If $\alpha =$ constant $\begin{cases} \omega_f = \omega_i + \alpha t \\ \theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2 \\ \omega_f^2 = \omega_i^2 + 2\alpha(\theta_f - \theta_i) \end{cases}$	If $a =$ constant $\begin{cases} v_f = v_i + at \\ x_f = x_i + v_i t + \frac{1}{2}at^2 \\ v_f^2 = v_i^2 + 2a(x_f - x_i) \end{cases}$
Work $W = \int_{\theta_i}^{\theta_f} \tau \, d\theta$	Work $W = \int_{x_i}^{x_f} F_x \, dx$
Rotational kinetic energy $K_R = \frac{1}{2}I\omega^2$	Kinetic energy $K = \frac{1}{2}mv^2$
Power $P = \tau\omega$	Power $P = Fv$
Angular momentum $L = I\omega$	Linear momentum $p = mv$
Net torque $\Sigma\tau = dL/dt$	Net force $\Sigma F = dp/dt$

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Rotating Rod

Saturday, 30 January, 2021 16:21

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.

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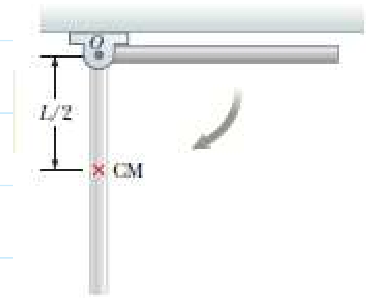
A uniform rod of length L and mass M is free to rotate on a frictionless pin passing through one end. The rod is released from rest in the horizontal position.



- What is its angular speed when the rod reaches its lowest position?
- Determine the tangential speed of the center of mass and the tangential speed of the lowest point on the rod when it is in the vertical position.

$$\Delta K + \Delta U = 0$$

$$\left(\tfrac{1}{2}I\omega^2 - 0\right) + \left(0 - \tfrac{1}{2}MgL\right) = 0$$

$$\omega = \sqrt{\frac{MgL}{I}} = \sqrt{\frac{MgL}{\frac{1}{3}ML^2}} = \boxed{\sqrt{\frac{3g}{L}}}$$

$$v_{\mathrm{CM}} = r\omega = \frac{L}{2}\omega = \boxed{\tfrac{1}{2}\sqrt{3gL}}$$

$$v = 2v_{\mathrm{CM}} = \boxed{\sqrt{3gL}}$$

Energy and the Atwood Machine

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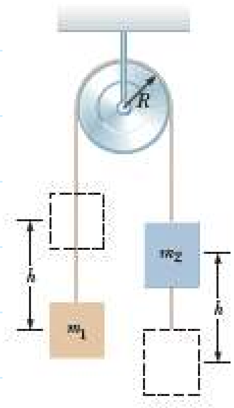
Two blocks having different masses m_1 and m_2 are connected by a string passing over a pulley. The pulley has a radius R and moment of inertia I about its axis of rotation. The string does not slip on the pulley, and the system is released from rest.

- Find the translational speeds of the blocks after block 2 descends through a distance h and
- Find the angular speed of the pulley at this time.



$$\Delta K + \Delta U = 0$$

$$[(\tfrac{1}{2}m_1 v_f^2 + \tfrac{1}{2}m_2 v_f^2 + \tfrac{1}{2}I\omega_f^2) - 0] + [(m_1 gh - m_2 gh) - 0] = 0$$

$$\tfrac{1}{2}m_1 v_f^2 + \tfrac{1}{2}m_2 v_f^2 + \tfrac{1}{2}I\frac{v_f^2}{R^2} = m_2 gh - m_1 gh$$

$$\tfrac{1}{2}\left(m_1 + m_2 + \frac{I}{R^2}\right)v_f^2 = (m_2 - m_1)gh$$

$$(1) \quad v_f = \boxed{\left[\frac{2(m_2 - m_1)gh}{m_1 + m_2 + I/R^2}\right]^{1/2}}$$

$$\omega_f = \frac{v_f}{R} = \boxed{\frac{1}{R}\left[\frac{2(m_2 - m_1)gh}{m_1 + m_2 + I/R^2}\right]^{1/2}}$$

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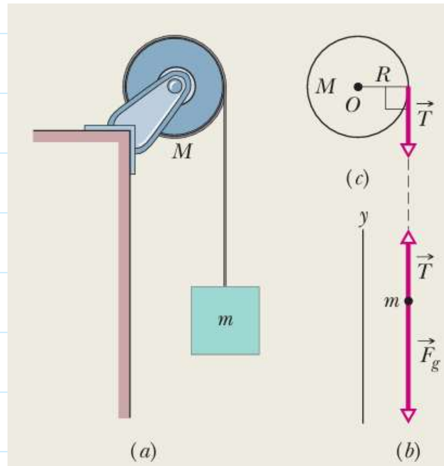
Let the wheel shown in the figure start from rest at time $t = 0$ and also let the tension in the cord be $6\ N$ and the angular acceleration of the disk be $-24\ rad/s^2$. What is its rotational kinetic energy at $t = 2.5\ s$?



Calculations: Because we want ω and know α and ω_0 $(= 0)$, we use Eq. 10-12:

$$\omega = \omega_0 + \alpha t = 0 + \alpha t = \alpha t.$$

Substituting $\omega = \alpha t$ and $I = \frac{1}{2}MR^2$ into Eq. 10-34, we find

$$K = \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}(\tfrac{1}{2}MR^2)(\alpha t)^2 = \tfrac{1}{4}M(R\alpha t)^2$$
$$= \tfrac{1}{4}(2.5\text{ kg})[(0.20\text{ m})(-24\text{ rad/s}^2)(2.5\text{ s})]^2$$
$$= 90\text{ J.} \qquad \text{(Answer)}$$

Calculations: First, we relate the *change* in the kinetic energy of the disk to the net work W done on the disk, using the work–kinetic energy theorem of Eq. 10-52 $(K_f - K_i = W)$. With K substituted for K_f and 0 for K_i, we get

$$K = K_i + W = 0 + W = W. \qquad (10\text{-}60)$$

Next we want to find the work W. We can relate W to the torques acting on the disk with Eq. 10-53 or 10-54. The only torque causing angular acceleration and doing work is the torque due to force $\vec{T}$ on the disk from the cord, which is equal to $-TR$. Because α is constant, this torque also must be constant. Thus, we can use Eq. 10-54 to write

$$W = \tau(\theta_f - \theta_i) = -TR(\theta_f - \theta_i). \qquad (10\text{-}61)$$

Because α is constant, we can use Eq. 10-13 to find $\theta_f - \theta_i$. With $\omega_i = 0$, we have

$$\theta_f - \theta_i = \omega_i t + \tfrac{1}{2}\alpha t^2 = 0 + \tfrac{1}{2}\alpha t^2 = \tfrac{1}{2}\alpha t^2.$$

Now we substitute this into Eq. 10-61 and then substitute the result into Eq. 10-60. Inserting the given values $T = 6.0$ N and $\alpha = -24$ rad/s^2, we have

$$K = W = -TR(\theta_f - \theta_i) = -TR(\tfrac{1}{2}\alpha t^2) = -\tfrac{1}{2}TR\alpha t^2$$
$$= -\tfrac{1}{2}(6.0\text{ N})(0.20\text{ m})(-24\text{ rad/s}^2)(2.5\text{ s})^2$$
$$= 90\text{ J.} \qquad \text{(Answer)}$$

Power

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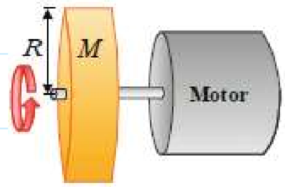
- [ ] H. D. Young and R. A. Freedman, *University Physics with Modern Physics*, 14th ed., PEARSON, 2016.
- [x] H. A. Radi and J. O. Rasmussen, *Principles of Physics For Scientists and Engineers*, 1st ed., SPRINGER, 2013.

A disk of mass $M = 0.2\ kg$ and radius $R = 5\ cm$ is attached coaxially to the massless shaft of an electric motor. The motor runs steadily at $900\ rpm$ and delivers $2\ hp$.



- What is the angular speed of the disk in SI units?
- What is the rotational kinetic energy of the disk?
- How much torque does the motor deliver?

Solution: (a) The angular speed of the motor of the disk is:

$$\omega = \left(900\frac{\text{rev}}{\text{min}}\right)\left(\frac{\text{min}}{60\ \text{s}}\right)\left(\frac{2\pi\,\text{rad}}{\text{rev}}\right) = 94.2\ \text{rad/s}$$

(b) The rotational kinetic energy of the disk is:

$$K_R = \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}\left(\tfrac{1}{2}M R^2\right)\omega^2$$
$$= \tfrac{1}{4}(0.2\ \text{kg}) \times (0.05\ \text{m})^2(94.2\ \text{rad/s})^2 = 1.11\ \text{J}$$

This is the amount of energy needed to bring the disk from rest to the angular speed $\omega = 94.2\ \text{rad/s}$.

(c) The power delivered by the motor to maintain a constant angular speed $\omega = 94.2\ \text{rad/s}$ for the disk and to oppose all kinds of friction is:

$$P = 2 \times (746\ \text{W}) = 1{,}492\ \text{W}$$

Using Eq. 8.41, $P = \tau\omega$, we can find the torque as follows:

$$\tau = \frac{P}{\omega} = \frac{1{,}492\ \text{W}}{94.2\ \text{rad/s}} = 15.8\ \text{m.N}$$